THEORY OF RELAXATION AUTOOSCILLATIONS OF A GAS-FLUIDIZED BED

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The physical nature of autooscillations of a quasifluidized bed which arise when there is a gas space under the gas distributor is explained and a model is constructed.

In addition to random pulsations of hydrodynamic parameters resulting from random local motions of particles and the fluidizing medium, the rupture of gas bubbles etc., which occur in nearly every quasifluidized bed, under certain quasifluidization conditions periodic motions characterized by a high degree of order are observed also. The study of such regular oscillations is not only of general scientific interest, but also is important for applications, since in a number of cases the use of organized beds with periodic pulsations offers certain technological advantages of improved heat transfer — a decrease in the volume of the bypassing fluidizing medium, etc.

Attempts have been made to explain these periodic motions by considering the interaction and merging of increasing small perturbations arising from the internal instability of the bed, with the formation of regular waves of finite amplitude which are damped or grow selectively as they propagate in the bed [1-3]. As a result the perturbations of the parameters of a quasifluidized bed acquire a quasiperiodic character and are described by a relatively narrow range of frequencies with one or more maxima. An alternative explanation is based on the assumption of the existence of synchronous coherent displacements of particles superimposed on their random pulsations [4, 5]. Such considerations are very useful in understanding the causes of the preferential generation of perturbations with a localized frequency spectrum, but they are clearly inadequate to describe the regular autooscillations sometimes observed.

Autooscillations which arise during the piston-like behavior of the initial quasifluidization as a result of cohesive forces between particles are well known [6, 7]. It is obvious also that regular oscillations in shallow beds are caused by the periodic rupture of large bubbles which occupy practically the whole cross section of the apparatus [8-10]. Autooscillations can arise also under more specific conditions, for example, in a bed of particles with adhesive films on their surfaces [11].

A distinctive autooscillation regime is observed also for a very broad range of gas flow rates which exceed the minimum quasifluidation rate when there is an empty space under the gas distributor plate, which is accessible to the gas, and the hydraulic resistance of the plate is much smaller than that of the bed itself. A preliminary physical analysis of such an effect, observed, for example, in [12, 13], was given by Davidson [14], but there is still no consistent theory of it. We discuss below a theoretical model of such autooscillations, paying particular attention to fundamentals. A detailed analysis of the results of the theory and a comparison with data from special experiments will be described in a later paper.

We consider a stationary bed of height H in an apparatus with a cross-sectional area S, containing a space of volume V under the plate. For simplicity, we assume that the total mass flux q of the gas entering the space is constant. Part of the gas filters through the bed at the filtration rate Q, and part remains in the space, causing a monotonic increase in the pressure p. The relation between Q and the pressure drop $p - p^0$ is shown by curve OAB of Fig. 1. For simplicity, we neglect the hydraulic resistance of the plate. At a pressure $p = p_0$ the weight of the particles of the bed is completely compensated by hydraulic forces (point A of Fig. 1) and the bed is partially fluidized in accord with the model in [7]. The resistance curve OAMC of such a bed

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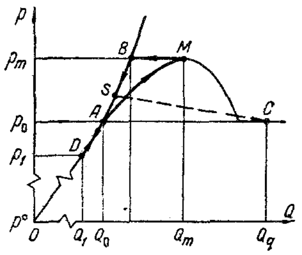

Fig. 1. Phase diagram of auto-oscillation cycle.

deviates from the curve OAB; its maximum is due to skin friction which is related to further compressive stresses on the walls of the apparatus as a result of the initial plastic deformation of the bed [7]. If the mass flux q of the entering gas is not too high so that the corresponding stationary flow rate Q_q of the gas in the bed is less than Q_m characterizing point M at the maximum of the actual quasifluidization curve, a stationary regime is established in which the flow of gas into the space is completely compensated by its outflow through the bed. In this case the bed is completely stationary if Q_q is less than the minimum quasifluidization rate Q_0; otherwise, it is partially fluidized in accord with the model in [7]. It is easy to show that such a regime is stable, and in accord with data in [12, 13] no pulsations arise. If the velocity of the entering gas is high so that $Q_q > Q_m$, the monotonic increase of pressure continues up to the maximum value p_m and the representative point of the system moves from its initial position D to M during the time T_1 as shown in Fig. 1.

It is clear that after the pressure in the space reaches its maximum value, equilibrium of the bed becomes impossible. The balance of the weight forces and skin friction, acting on the lower part of the bed when it is stationary in state M, and the hydraulic forces is disrupted. As a result, the bed is separated from the supporting plate and enters a dynamic state. This transition occurs practically instantaneously. Actually, the characteristic time to accelerate all particles of the bed is determined by the speed of an elastic compressional wave, which is of the same order of magnitude as the speed of sound in the stationary bed. During this time the lower part of the bed moves upward as a piston and captures the fluidized particles of the upper part so that the whole layer rapidly approaches a state of dense packing. This time is short in comparison with the period of an autooscillation cycle, generally amounting to a fraction of a second [13]. Thus we can assume that the transition to the dynamic state is a jump at constant pressure in the space (jump M → B in Fig. 1). For simplicity, we neglect possible differences between resistance curves of the initial stationary and moving bed which may be relatively broken up.

As the bed moves upward some of the particles fall down from its lower surface just as during the piston regime of the initial quasifluidization [6, 7]. However, the time scale of the falling process, which is also the scale of the uniform broadening of the bed, is the same as the characteristic lifetime of the piston, amounting to several seconds, and is appreciably longer than the period of autooscillations. This is important in the context of this paper. Therefore, in investigating autooscillations such falling can be neglected in the first approximation. The concept of piston-like motion of a bed agrees with experimental data in [13] and with the hypotheses in [14].

Hydraulic forces acting on a piston separated from the plate are partially compensated by gravitational forces on the particles and skin friction, which is considerably weaker than static friction. The excess of the hydraulic forces, characterized by the pressure drop $p - p_0$, is compensated, according to Newton's second law, by the inertial force hindering the acceleration of the piston. As a result of the upward displacement of the bed the gas in the space can enter the lower part of the apparatus containing the bed. The gas expands, its pressure drops, and the representative point of the system moves downward along curve OAB of Fig. 1.

Analysis shows that there is a certain stationary state of the piston corresponding to its motion with constant velocity; this state is represented by point S of Fig. 1. We do not take account of the falling down of particles and the related spreading out of the piston. If the piston moves slowly, i.e., if the gas is not expanding very rapidly, the transition to the stationary state occurs aperiodically along the path B → S of Fig. 1. The corresponding increase in velocity of the piston to the limiting value w_s is shown in Fig. 2. If the acceleration of the piston is large enough the pressure of the gas in the space may become smaller than p_0 so that the piston, continuing to move upward by inertia, begins to slow down. At a certain instant its velocity changes sign and it begins to descend. The pressure in the space increases

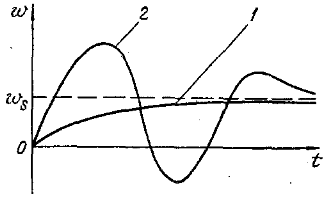

Fig. 2. Characteristic time dependence of piston velocity for aperiodic (1) and oscillatory (2) regimes.

and as soon as it exceeds the value p_0 the hydraulic forces begin to slow down its fall and the piston again moves upward and the process is repeated. In this case the approach to the steady state is oscillatory in character so that the representative point of the system, located initially at B, oscillates along curve OAB and approaches S alternately from above and from below. These oscillations are damped by the small force of dynamic friction and after a certain time the oscillatory regime becomes aperiodic. The characteristic time dependence of w corresponding to the oscillatory regime is also shown in Fig. 2.

If the approach to the steady state is aperiodic or the amplitude of the oscillations is so small that the lower surface of the oscillating piston does not again come in contact with the gas distributor plate, autooscillations are impossible. In this case the falling down of the particles leads finally to a broadening of the piston and to a steady quasifluidized state with an average porosity corresponding to the flow rate Q_q. The transition to this state is shown by the dashed arrow in Fig. 1. Suppose now the conditions are such that the amplitude of the oscillations is large and the first autooscillation cycle is interrupted by the impact of the piston on the plate. If the impulse of the collision is large enough to produce compressive stresses causing a maximum on the actual quasifluidization curve, the process described above will be repeated; i.e., autooscillations begin. The total period of the autooscillation cycle is made up at the periods T_1 and T_2 during which the bed is in static and dynamic states, respectively. It is clear that such autooscillations are of the relaxation type; the form of the autooscillation cycle is shown by the closed curve DMBD on Fig. 1.

Thus autooscillations are related, first, to the existence of free energy stored as energy of compression of the gas in the space, second, to the possibility of a sufficiently rapid transformation of this energy into kinetic energy of the piston, and, third, to the presence of additional skin friction forces which do not disappear in the complete compensation of the weight of the bed by hydraulic forces. This makes it possible to understand why such autooscillations are not observed in beds fluidized by incompressible liquids when there is a small space and a high resistance of the gas distributor plate [12,13].

A quantitative theory of autooscillations can be based on the physical picture described above. The mass of the gas under the bed and the equation describing its accumulation are given by the expressions

$$m = \rho (V + Sz), \quad dm/dt = q - \rho SQ. \tag{1}$$

The relation between the pressure drop in the bed and the rate of flow of gas in it, the equation of state of the gas, and the condition defining the thermodynamic nature of the expansion and compression processes (for example, the condition that these processes are isothermal or abiabatic) can be written formally as

$$p - p^0 = H\varphi(Q), \quad p = p(\rho, T), \quad F(\rho, p, T) = \text{const.} \tag{2}$$

The equation of motion of the piston in the dynamic stage has the form

$$\rho' SH \frac{d^2z}{dt^2} = \rho' SH \frac{dw}{dt} = S(p - p^0) - \rho' SHq - LHf(w). \tag{3}$$

Equations (1)-(3) form a complete system for determining the unknown gas parameters ρ, p, and T, the velocity w and the z coordinate of the lower surface of the piston, the flow rate Q of the gas in the bed, and the mass of the gas m. In principle, this system can easily be investigated numerically for the most varied relations $\varphi(Q)$ and f(w) and for various functions $p(\rho, T)$ and $F(\rho, p, T)$ in both the static and dynamic stages of the autooscillation cycle. Here we restrict ourselves to obtaining only qualitative results in analytic form which hold approximately for an important special case. We assume that the gas is ideal and that its specific volume varies isothermally, so that we can take F = T. The function $\varphi(Q)$ is assumed

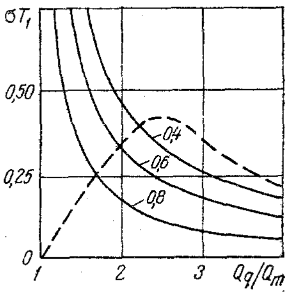

Fig. 3. Dependence of σT_1 from (8) on Q_q/Q_m for various Q_1/Q_m (numbers on curves). The open curve shows the characteristic curve taking account of the dependence of Q_1 on Q_q.

linear for a stationary bed. This assumption is valid for a bed of fine particles where in calculating the static stage of the autooscillation cycle we neglect for simplicity the difference between the resistance curves of Fig. 1 for stationary and partially fluidized beds. We assume that the maximum pressure drop $p_m - p^0$ in the bed is small in comparison with the pressure p^0 of the gas above the bed and the volume Sz of the empty space in the apparatus is small in comparison with the volume of the space V. Finally, we neglect dynamic friction which usually is small in comparison with the hydraulic forces. It is easy to see that giving up these simplifications which enable us to linearize the problem does not complicate it in principle, but only leads to a certain number of computational difficulties.

Thus, we assume further

$$p(\rho, T) = \rho RT/M, \quad F=T, \quad \varphi(Q) = kQ, \quad Sz \ll V,$$
$$p_m - p^0 \ll p^0, \quad f(w) \ll (S/LH)(p_m - p^0). \tag{4}$$

Let us consider first the static period of the autooscillation cycle when z = 0 and w = 0. Using (2) and (3) and taking $p \approx p^0$ in accord with (4), we obtain the following problem for the total pressure drop $p - p^0$:

$$\frac{d(p-p^0)}{dt} + \frac{Sp^0}{kHV}(p-p^0) = \frac{RT}{MV} q, \quad p-p^0|_{t=0} = (\Delta p)_1 = p_1 - p^0. \tag{5}$$

The solution of this problem has the form

$$\Delta p = p - p^0 = (\Delta p)_q - [(\Delta p)_q - (\Delta p)_1] e^{-\sigma t}, \tag{6}$$

where we have introduced the quantities

$$(\Delta p)_q = kHQ_q, \quad Q_q \approx Q_q^0 = \frac{q}{S\rho^0} = \frac{RT}{M} \cdot \frac{q}{Sp^0}, \quad \sigma = \frac{Sp^0}{kHV}. \tag{7}$$

The duration of the static stage is found by solving Eq. (6) for t when $\Delta p = (\Delta p)_m = p_m - p^0$, i.e.,

$$T_1 = \frac{1}{\sigma} \ln \frac{(\Delta p)_q - (\Delta p)_1}{(\Delta p)_q - (\Delta p)_m} = \frac{kHV}{Sp^0} \ln \frac{1 - Q_1/Q_q}{1 - Q_m/Q_q}. \tag{8}$$

The last of Eqs. (8) is obtained by using (7) and determining the various pressure drops in terms of the corresponding flow rates (cf. Fig. 1). Equation (8) has the same structure as the equation given in [13]. It is clear that the length of the static period increases linearly with the height of the bed, the volume of the gas space under it, and the coefficient of hydraulic resistance (i.e., actually for a decrease in the size of the particles for a given porosity of the bed) and decreases inversely proportionally to the absolute pressure of the gas and the cross-sectional area of the apparatus. The value of T_1 depends on the total flow rate of the gas in a more complicated way; as Q_q increases, the time for the bed to reach a stationary state decreases to zero. The dependence of T_1 on Q_q for various values of Q_1/Q_m is shown in Fig. 3; we note that the experimental value of this ratio [13] is 0.7.

Let us consider now the dynamic stage of the autooscillation cycle. The parameters associated with the stationary regime (point S in Fig. 1) are determined by equations following from (1)-(4);

$$\frac{p_s}{\rho_s} = \frac{RT}{M}, \quad p_s - p^0 = \rho' Hg = kHQ_s, \quad \rho_s S w_s = q - \rho_s S Q_s. \tag{9}$$

Thus, using the approximations indicated, in particular, neglecting dynamic friction, the quantities ρ_S, p_S, and Q_S are the same as ρ_0, p_0, and Q_0 characterizing the state of minimum quasifluidization (point A of Fig. 1).

In the case considered, Eqs. (1)-(3) reduce to the form

$$\frac{d^2z}{dt^2} = \frac{dw}{dt} = g\left(\frac{Q}{Q_0} - 1\right), \quad \frac{dm}{dt} = \rho_0 S Q_0 \left(\frac{\rho^0 Q_q}{\rho_0 Q_0} - \frac{\rho Q_0}{\rho_0 Q_0}\right),$$
$$\frac{p - p^0}{p_0 - p^0} = \frac{Q}{Q_0}, \quad \rho = \frac{M}{RT} p, \quad m = \rho(V + Sz), \tag{10}$$

where we have used (4) and (9).

Using inequality (4) and retaining higher order terms in Sz/V and $(p - p^0)/p^0$ we obtain from (10)

$$\frac{d^2w}{dt^2} + 2\alpha \frac{dw}{dt} + \beta w = \gamma, \quad (\Delta p)_0 = p_0 - p^0,$$
$$\alpha = \frac{1}{2} \frac{SQ_0}{V} \cdot \frac{p^0}{(\Delta p)_0} = \frac{1}{2} \cdot \frac{Sp^0}{kHV} = \frac{\sigma}{2}, \tag{11}$$
$$\beta = 2\alpha \frac{g}{Q_0}, \quad \gamma = 2\alpha g \left(\frac{Q_q}{Q_0} - 1\right).$$

By measuring time from the start of the dynamic period the corresponding initial conditions have the form

$$w|_{t=0} = 0, \quad \left.\frac{dw}{dt}\right|_{t=0} = \frac{p_m - p_0}{\rho' H} = a, \tag{12}$$

where a is the initial acceleration of the piston.

The solution of problem (11), (12) depends on the sign of $\alpha^2 - \beta$. The aperiodic regime occurs if this quantity is positive and the oscillatory regime, if it is negative. Thus, the condition for oscillations to occur has the form [cf. the definitions of α and β in (11)]

$$\frac{(\Delta p)_0}{p^0} > \frac{1}{4} \frac{SQ_0^2}{Vg}. \tag{13}$$

When this condition is satisfied we obtain from (11) and (12)

$$w = w_s(1 - e^{-\alpha t}\cos\omega t) + \omega^{-1}(a - \alpha w_s)e^{-\alpha t}\sin\omega t,$$
$$w_s = \frac{\gamma}{\beta} = Q_q - Q_0, \quad \omega^2 = \beta - \alpha^2 = \frac{2\alpha g}{Q_0}\left(1 - \frac{\alpha Q_0}{2g}\right). \tag{14}$$

The displacement of the lower surface of the moving piston is

$$z = \int_0^t w\,dt = w_s t + \frac{a}{\beta}(1 - e^{-\alpha t}\cos\omega t) - \frac{\alpha}{\omega\beta}\left[a - \alpha w_s\left(1 - \frac{\omega^2}{\alpha^2}\right)\right]e^{-\alpha t}\sin\omega t. \tag{15}$$

The duration T_2 of the dynamic stage of the autooscillation cycle is obviously given by the smallest positive root of the equation

$$z|_{t=T_2} = 0. \tag{16}$$

The condition for the existence of such a nonzero root, actually imposed on the constant parameters of the process, is the necessary condition that the separated piston again come in contact with the gas distributor plate. This is a considerably more restrictive condition than (13). The velocity of the piston directly before contact is determined from (14) for $t = T_2$. It determines the energy which in principle can be expended in the reconsolidation of the bed, i.e., on the rearrangement of its structure with the creation of compressive stresses. However, a quantitative relation between this energy and the value of the compressive stresses determining the excess pressure drop $p_m - p_0$ remains unclear in the absence of an adequate structural model of free-flowing media. Thus, the indicated pressure drop must be considered as a certain a priori quantity to be determined by experiment.

The initial flow rate of the gas Q_1 in the static stage entering (8) is determined from a relation following from (10):

$$Q_1 = Q_0 + \frac{Q_0}{g} \cdot \left.\frac{dw}{dt}\right|_{t=T_2}. \tag{17}$$

A complete investigation of Eqs. (14)-(17) for various values of the parameters is a very cumbersome calculational problem and may comprise the subject of an independent investigation. Here we consider only two of the simplest, although somewhat formal, limiting cases. Let

$$a \ll \omega\gamma/\beta, \quad \omega \sim \sqrt{\beta} \gg \alpha,$$

which can be satisfied, for example, for large Q_q and small $p_m - p_0$. In this case we have from (14) and (15)

$$w \approx w_s(1 - \cos\omega t), \quad z \approx w_s(t - \omega^{-1}\sin\omega t),$$

so that Eq. (16) in general has no positive roots. If oscillations arise, i.e., if Eq. (13) is satisfied, they stop after a time comparable with the characteristic time of broadening of the bed. The latter agrees with data in [13], where degeneracy of the autooscillations was noted for increasing rate of inflow of gas q, and also shows that $p_m - p_0$ plays a very important role in the generation of the autooscillation regime. The relation of this regime to the presence of a maximum on the actual quasifluidization curve is emphasized in [13].

Suppose now

$$a \gg \omega\gamma/\beta, \quad \omega \sim \sqrt{\beta} \gg \alpha,$$

which is possible for large $p_m - p_0$ and values of Q_q slightly different from Q_m. Then

$$w \approx \frac{a}{\omega}\sin\omega t, \quad z \approx \frac{a}{\omega^2}(1 - \cos\omega t),$$

so that Eq. (16) gives

$$T_2 \approx \frac{2\pi}{\omega} \approx 2\pi\left[\frac{V}{Sg} \cdot \frac{(\Delta p)_0}{p^0}\right]^{1/2} = 2\pi\left(\frac{\rho' HV}{S' p^0}\right)^{1/2}. \tag{18}$$

Thus, in this case there can be autooscillations which increase proportionally to the square roots of the volume of the space, the height and effective density of the bed, and inversely proportionally to the square roots of the gas pressure and the cross-sectional area of the apparatus. In this case, the autooscillations have a dynamic character, and it is clear from (17) that $Q_1 \approx Q_m$, so that point D on Fig. 1 is very close to M and the static period is very short. This conclusion is very natural for the assumptions used. Actually, dynamic friction and the dissipation of energy in the compression or expansion of the gas have been neglected. Under these conditions the only physical cause of damping of the vibrations is the phase difference between the pressure oscillations (i.e., the force on the piston) and the velocity of the piston which is proportional to α, which was allowed to approach zero in the derivation of (18). The example presented shows that ignoring the dependence of Q_1 on Q_q, as was done, for example, in [13], is inadmissible. Thus, it is easy to show that the actual relation between T_1 from (8) and Q_q will be represented by curves of the type shown dashed in Fig. 3.

The frequency ν of autooscillations is equal to $(T_1 + T_2)^{-1}$. Using (8) and (18) we conclude that the general character of the dependence of ν on V, H, and other parameters is confirmed by the experimental data, particularly those from [13].

The autooscillation regime investigated can be regarded as a peculiar new mechanism of the initial quasifluidization, differing in principle from the four basic mechanisms pointed out in [7].

In conclusion, we note that the theory presented above can easily be generalized to situations which are less restrictive than those considered here. The only serious restriction is the assumption that the falling down of particles and the formation of bubbles within the moving piston can be neglected. This requires further investigation.

NOTATION

a, initial acceleration of piston; F, thermodynamic potential in (2); f, force of dynamic friction per unit area; g, acceleration due to gravity; H, height of bed; k, coefficient of hydraulic resistance; L, perimeter of cross section of apparatus; M, molecular weight of gas; m, mass of gas under bed; p, pressure; Q, volume flow rate of gas in bed (filtration rate); Q_q, parameter in (7); q, total mass of gas entering per unit time; R, gas constant; S, cross-sectional area of apparatus; T, temperature; T_1, T_2, lengths of static and dynamic periods of autooscillation cycle; t, time; V, volume of space under plate; w, velocity of piston; z, coordinate of lower surface of piston; α, β, γ, constants in (11); ν, frequency of autooscillations; ρ, density of gas; ρ' effective density of bed; σ, parameter in (7); φ, hydraulic resistance per unit volume; ω, parameter in (14). Indices: 1, 0, s, m, initial state of the bed in the static stage, the state of minimum quasifluidization, the stationary state, and the state with the maximum pressure drop, respectively; o, state of the gas above the bed.

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COOLING OF A COARSE LUMP IN A BED OF "FINE" PARTICLES

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The cooling of a coarse lump having the form of a rectangular prism in a blow-through bed of "fine" particles is discussed. A solution is obtained for large and small values of Fo by using Laplace transforms.

The heating or cooling of a polydisperse bed of lumps is a frequently occurring practical problem. Large lumps have many through pores so that a gaseous medium not only flows around a coarse lump, but also filters through its pores and increases the heat transfer.

Thus, the physical problem is the following. A bed through which air filters contains a coarse lump at a certain depth. At zero time the whole bed, including the lump, is heated to the temperature t_0 and is cooled by air with a temperature T' at the inlet to the bed. It is required to find the time to cool the coarse lump to a given temperature.

The following assumptions and simplifications are made.

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